CALIBRATION OF DISPERSION MODELS USING MU RIVER, BENUE STATE, NIGERIA

I.M. Aho, G.D. Akpen and S.J. Uungwa

Abstract- The research was aimed at Calibrating dispersion models using Mu River. The study was effected by a means of tracer-dye. Experimental data from tracer injections as well as data from hydraulic parameters were used to calculate the dispersion coefficient of River Mu in Makurdi, Benue state, Nigeria.The work was carried out at the peak of rainy season. Data for the study was gotten from injection of 1 kg of soluble sulphur black (BR) dye. Dispersion coefficient as a fundamental parameter in hydraulic modelling in river pollution was estimated using three models namely: Agunwamba, Leverspiel and Smith as well as Deng et al. The first gave 42 m^2/s. The second model gave 17 m^2/s and the third, 45 m^2/s. The values of dispersion coefficient obtained using Agunwamba model and Deng et al model were adopted because of the closeness and fair comparison with the values of dispersion coefficient in literature. It was observed that, dispersion coefficient values are affected by parameters like river velocity, hydraulic depth and cross–sectional width of the river.

Keywords: Calibration, dispersion coefficient, dispersion models, pollution simulation, Slug injection.

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1 Introduction

Dispersion may be taken as the distribution of pollutants in surface and groundwater systems [1]. Dispersion is therefore mixing caused by physical processes. Dispersion in a river system helps to reduce the local pollution level considerably by distributing the dissovled substances gradually with time, there by changing the concentration of the substances which are present in the water body. Moreover, it is one of the most important factors for evaluating the pollutant behavior in a river or stream [2]. Dispersion process is important in water quality management and pollution control, and determines the capacity of a stream to assimilate contaminants. If the capacity of a stream to assimilate contaminants is over-estimated, serious pollution can occur. Under estimating can lead to under utilization of the stream.This would involves more expenditures in treatment facilities. Adequate prediction of waste concentration downstream from a waste discharge position enables the Engineers to design more rationally

the outflow. Dispersion studies are also very relevant in the determination of re-aeration capacity of streams. The extent of dispersion is quantified by the dispersion coefficient, D or its dimensionless

number, dispersion number, ∂ which is the inverse of the pellet number which has been used widely in chemical reactor engineering.

The response to the slug injection of a soluble tracer is assumed to imitate the characteristics of a soluble pollutant, so understanding tracer mix and disperse in a stream is essential to understanding their application in simulating pollution [3]. This is the case in either a steady flowing river or in the unsteady oscilatory stage and flow of a tidal estuary. Measured tracer-response curves produced from injection of a known quantity soluble tracers provides an efficient method of obtaining the data necessary to calibrate and verify pollutant transport models. Extensive use of flourescent dyes as water tracers to quantify the transport and dispersion in rivers and streams began in the early to mid–1960s [4].

Dye study is one of the most reliable means to estimate dispersion coefficient [5]. According to [6] dye is introduced into the river site and measurements of the dye concentration are made at several locations (distance) downstream from the point of injection. It can also impact negatively on water especially when present in significant concentration [7]. Because dispersion coefficient is dependent on the velocity profile of a river, it is then a function of the river flow rate. Therefore, a dispersion coefficient computed by a tracer–dye study for one flow rate segment of the river will not apply to a situation of another river segment of different flow rate .In such instance, predictions may be made from the results of one dye study or a series of dye studies may be performed [4].

A large number of researchers have contributed to the understanding of the mechanism of dispersion in rivers, beginning with the simplest dispersion of dissovled contaminations in pipe flow. Later the concept of dispersion was extended to the mixing in open channels and further to natural streams. Many theoretical and emperical formulations or models have been proposed to determine the disperion coefficient. The earlier ones include those proposed by [8], [9], [10] and [11] [12].

Dispersion coefficient cannot be measured directly. Physical measurements of some parameters are required. The dispersion coefficient, D or the dimensionless equivalent called dispersion number, ∂ can be estimated by conducting tracer experiment or predicted from emperical equations [13]. Tracer experiments could be more reliable because they provide on the spot assessment with minimum assumptions.

The research objective evaluated the dispersion coefficient of MU river using three models namely; Agunwamba, Leverspiel and Smith as well as Deng *et al.* The study ascertained the models that fit the dispersion profile of the study area. The data for the research was obtained from tracer experiment conducted on Mu riv er in Benue State, Nigeria.

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2 Methodology

2.1 Study area

The study was carried out on a small section of Mu river along the TyoMu-Jagu stretch. Mu river is a tributary of River Benue; a major River in Makurdi Metropolis, Benue state and it originates from Ikpa Agule, Mbatierev, Gboko Local government Area of Benue State (Fig 1). It confluences with another smaller river called Adebe just before Benue Breweries Limited (BBL) plant along Makurdi Gboko road, thereby expanding its volumetric flow (Fig 2). The people in Mu settlement and its environs depend solely on Mu river for their agricultural, domestic and other applications. Thus, the need for calibration of dispersion models for the river and quality assesment.

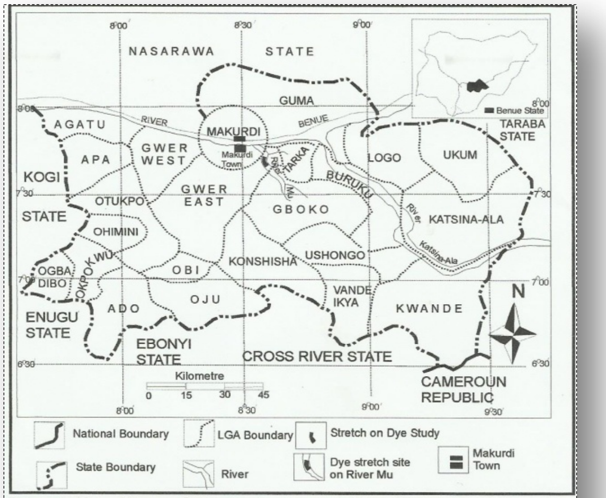

Figure 1: Map of Benue State showing the study River (River Mu).
Source: Ministry of Lands and Survey Makurdi

2.2 Sampling points

Fig. 1. Map of Benue State showing the study River (River Mu)
Source: Ministry of Lands and Survey, Makurdi.

Ten sampling points, approximately ne ve sampling. The sampling points were close to human settlements for safety reasons but away from the immediate and direct access of human activities

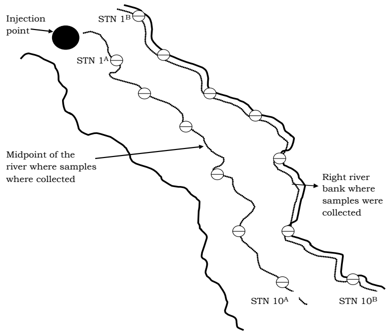

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Fig. 2. A Segment of Mu River showing the Main Sampling Stations (STN) along the River Centerline and the other Sampling Stations near the River Bank.

2.3 Field measurement and sampling

The dimensions of the river were obtained with a calibrated rod and surveyors tape. The flow velocity was determined by float method; using stopwatch and surface float (cork). Sulphur Black (BR) dye was used to carry out this study. One kilogram (1 kg) of the tracer (dye) was instantaneously introduced at a point, 50 m from the first sampling point. After thorough mixing, an initial concentration of the tracer in the river was collected before the actual sampling commenced. Samples were then collected at the 10 sampling points at 3 minutes (180 seconds) intervals. The injection of tracer and sampling was carried out on a paddled boat to minimize disturbance of the river flow pattern.

2.4 Laboratory analysis

The samples were collected with sterilized plastic containers labeled 1 to 10 on the same day. These samples were transferred immediately into a low-temperature chamber and taken to the Civil Engineering Laboratory, University of Agriculture Makurdi, Benue State, Nigeria for analysis. The analysis performed on the samples was to determine the coresponding concentration of the tracer in the water as it flows down stream. A UV–spectrophotometer was used in determing the dye concentration of the sample according to tracer-dye study guidelines of Alberta Environment [14].

2.5 Model calibration

The data gotten from both field and laboratory studies were subjected to mathematical modelling as a comparative study; Agunwamba, Leverspiel and Smith, Deng models, were adopted to test the experiemental data. The aforementioned models are hence applied to the study.

The relationship between variance and dispersion is derived analytically by using statistical moment method based on [15].

$$\partial=\frac{1}{8}\left(\sqrt{8\sigma^2+1}-1\right) \qquad (1)$$

While σ^2 is the normalized variance which is computed from constant length variable time tracer experiment and is given by:

$$\sigma^2=\left[\frac{1}{\bar{\theta}}\right]^2\left[\frac{\sum_{i=1}^{n}c_i t_i^2}{\sum_{i=1}^{n}c_i}-\bar{\theta}^2\right]. \qquad (2)$$

In which t, is the time after injection of tracer (seconds), C = tracer response concentrations at the exit stream (mg/L); θ is the average flow time [16] given by:

$$\theta=\frac{\sum_{i=1}^{n}c_i t_i}{\sum_{i=1}^{n}c_i} \qquad (3)$$

If the variable distance – variable time approach is employed in the tracer experiment, the corresponding equations for ∂ and σ^2 as derived by [13]. Is as shown in equation (4)

$$\partial=\frac{1}{29.2}\left(\sqrt{1+15\sigma^2}-1\right) \qquad (4)$$

Where:

$$\sigma^2=\frac{\sum_{i=1}^{n}\left(\frac{T}{1-\xi}\right)^2 c}{\sum_{i=1}^{n}c}-\left[\frac{\sum_{1=1}^{n}\left(\frac{T}{1-\xi}\right)c}{\sum_{i=1}^{n}c}\right]^2 \qquad (5)$$

Where the summation is taken over all the uniformly spaced readings. The parameters $T=t/\theta,\xi=\frac{X}{L}$, L is the channel length, x is the distance from the outlet and t is the time after tracer injection. The dispersion coefficient, D is given by:

$$D=u\partial L \qquad (6)$$

Where: u = average river distance velocity
L = River distance
∂ = dispersion number

The dispersion coefficient of Mu River can also be determined based on geomorphologic parameters of the River as presented by [17] as given in equation (7).

$$D=\frac{0.15}{8\xi}\left[\frac{w}{h}\right]^{1.67}\left[\frac{u}{u_*}\right]^2(2) \qquad (7)$$

$$\xi=0.145+\left[\frac{1}{3520}\right]\left[\frac{u}{u_*}\right]\left[\frac{w}{h}\right]^{1.38} \qquad (8)$$

Shear velocity,

$$u_*=\sqrt{gds} \qquad (9)$$

3 Data Analysis

3.1 Calibration of Agunwamba (2001) model

Using equation (4), (5) and computed values

from table 1, we have:

$$\sigma^2 = \frac{32610.0982}{1553.7} - \left[\frac{6482.5344}{1553.7}\right]^2$$

$$\sigma^2 = 20.9887 - 17.4083 = 3.58$$

$$\partial = \frac{1}{29.2}\left(\sqrt{1+15(3.58)}\ \text{-}1\right)$$

$$\partial = 0.0342\left(\sqrt{1+15(3.58)} - 1\right)$$

$$\partial = 0.0342\left(\sqrt{54.7} - 1\right)$$

0.0342 x 6.3959 = 0.2187

∂ = 0.2187

From equation (6), coefficient, D is obtained.

D = 0.38 x 500 x 0.2187

D = 41.55 m^2/s

Table 1: Computed Table for Dispersion Number using Agunwamba

(m) Distance	Time (Sec), t	Midpoint conc. (Mg/L), C	Ct	T=t/θ	$\xi = \frac{X}{L}$					
50	180	250.60	45108	0.3211	0					
100	360	340.30	122508	0.6422	0.1	0.9	6.422	41.1242	2185.4066	13994.5655
150	540	347.40	187596	0.9633	0.2	0.8	4.817	23.2035	1673.4326	8060.8959
200	720	349.80	251856	1.2844	0.3	0.7	4.281	18.3270	11497.498	6410.7840
250	900	152.60	137340	1.6056	0.4	0.6	4.014	16.1122	612.5364	2458.7217
300	1080	99.50	107460	1.9267	0.5	0.5	3.8534	14.8487	383.4133	1477.4557
350	1260	5.90	7434	2.2478	0.6	0.4	3.7463	14.0348	22.1032	82.8053
400	440	4.40	6336	2.5689	0.7	0.3	3.6699	13.4616	16.1476	57.4209
450	1620	2.60	4212	2..8900	0.8	0.2	3.6125	13.0502	9.3925	33.9305
500	1800	0.6	1080	3.2111	0.9	0.1	3.5679	12.7299	2.1407	7.6794
	$\sum$	1553.7	870930				38.3042	166.9952	6482.5344	32,610.0982

3.2 Calibration of Leverspiel and Smith (1957) Model

Using equation (1), (2) and computed values from table 2, we have;

$$\sigma^2 = \left[\frac{1}{560.55}\right]^2 \left[\frac{601{,}781{,}400}{1553.7} - 560.55^2\right]$$

$$\sigma^2 = [0.000603183][73105.1881]$$

$$\sigma^2 = 0.000603183 \times 73105.1881 = 0.2369$$

$$\sigma^2 = 0.2369$$

$$\partial = 0.125\left(\sqrt{8 \times 0.2369} + 1 - 1\right)$$

$$\partial = 0.125\left(\sqrt{2.8952 - 1}\right)$$

∂ = 0.08769

From equation (6), the dispersion coefficient, D is obtained.

D = 0.08769 x 0.38 x 500 = 16.66 m^2/s

D = 17 m^2/s

Table 2 : Computed Table For Dispersion Coefficient usi

Station	Distance (m)	Midpoint conc. (mg/L), C	Time (Sec) t	Ct
1	50	250.60	180	45108
2	100	340.30	360	122508
3	150	347.40	540	187596
4	200	349.80	720	251856
5	250	152.60	900	137340
6	300	99.50	1080	107460
7	350	5.90	1260	7434
8	400	4.40	1440	6336
9	450	2.60	1620	4212
10	500	0.6	1800	10801
	$\sum$	1553.7	9900	870930

3.3 Calibration of Deng *et al.*, Model

Applying equation (7), (8) and 9)

Acceleration due to gravity, g = 9.8 m/s^2

d = River depth = 4.94 m

w= Average width of the river=18.54.

Slope of MU River, which was obtained as 0.00006 from the topographic map of the area.

:.

$$u_* = \sqrt{9.8 \times 5.94 \times 0.00006} = \sqrt{0.00349272}$$

$$u_* = 0.0591$$

$$\xi = 0.145 + 0.000284 \left[\frac{0.38}{0.0591}\right] \left[\frac{18.54}{4.94}\right]^{1.38}$$

$$\xi = 0.145 + 0.000284 \times 6.4298 \times 6.2037$$

$$\xi = 0.1563$$

$$D = \frac{0.15}{8(0.1563 \left[\frac{18.54}{4.94}\right]^{1.67}} \left[\frac{0.38}{0.0591}\right]^2$$

$$\frac{0.15}{1.2504} \times 9.1037 \times 41.3421$$

$$0.1199 \times 9.1037 \times 41.3421$$

D= 45 m^2/s

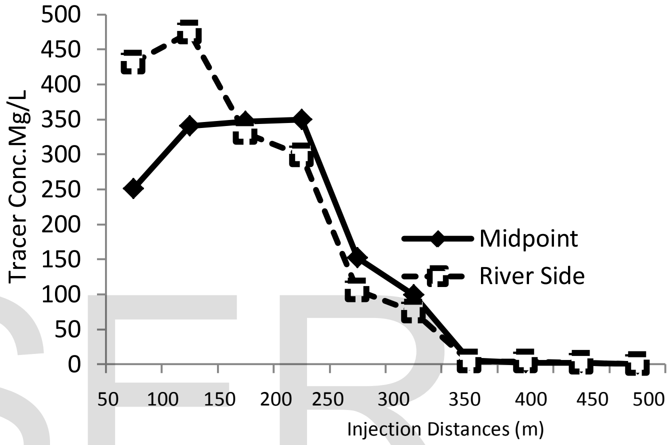

Fig. 3. Measured Tracer Concentration at the Midpoint and near the Right Bank of the River against Injection distances.

4 Results and Discussions

4.1 Field and laboratory results

The Dispersion data of MU river is presented in table 1, and the plot of tracer concentrations at midpoint and right side of the river bank against injection distances are as shown in Fig. 3

Table 3**:** Mu River Dispersion Data

Stations	Distance (x) (m)	Tracer conc. at the station (mg/L)		Time after release of tracer in sec.	Av. Cross sectional Area,A_x (m^2)	Witdth (m)	Depth (m)	Ave vel. (m/s)	Mean discharge (m^3/s)
		Midpoint	Right Side						
0	0	19000	-	0	99.545	21.50	4.63	0.40	39.82
1	50	250.60	430.70	180	149.52	24.00	6.23	0.39	58.31
2	100	340.30	474.10	360	80.52	18.30	4.40	0.37	29.79
3	150	347.40	330.50	540	69.58	14.20	4.90	0.41	28.53
4	200	349.80	298.60	720	79.98	15.50	5.16	0.42	33.59
5	250	152.60	105.30	900	48.88	13.00	3.76	0.40	19.55
6	300	99.50	75.40	180	198.75	26.50	7.50	0.35	69.56
7	350	5.90	4.20	1260	167.64	25.40	6.60	0.36	60.35
8	400	4.40	3.80	1440	36.36	18.60	2.02	0.38	13.82
9	450	2.60	2.40	1620	43.20	12.00	3.60	.0.36	15.55
10	500	0.60	0.60	1800	85.25	15.50	5.50	0.34	28.99
Average					91.58			0.38	34.80

Table4: Summary of Dispersion coefficients using different models.

S/No	Models	∂	D (m^2/s)	U (m/s)	θ (Hrs)
1	Agunwamba	0.2187	42	0.38	560.55
2	Leverspiel and Smith	0.08769	17	0.38	560.55
3	Deng *et al*	-	45	0.38	-

It can be observed from the curve above that the concentration of the tracer reduces gradually from the injection point to the furthest distance down stream (500 m). It therefore means that the effect of this tracer (dye) may not be felt or experienced at an unknown distance down stream. The loss of tracer in transit can be attributed to adsorption of sediments (silt and clay), adhesion on sediments, Photochemical decay and reactions [18]. The concentration of the tracer was seen to have been higher at the middle than the side of the of the river at the injection point. This is however, different down stream as the concentration of the tracer at the side of the river increased abdruptly before falling. This could be due to turbulence and wind effect.

4.2.2 Dispersion coefficient, D

Dispersion Coefficient D is a fundamental parameter in hydraulic modeling. Hence its importance in model calibration cannot be over-emphasized. Dispersion Coefficient values as obtained using Agunwamba and Deng *et al* model were favourably close as it gave the values 42 m^2/s and 45 m^2/s respectively. However, the value of dispersion coefficient gotten from the use of Leverspiel and Smith model was a deviation from erstwhile values

The reason for this disparity is because Leverspiel and smith model is based on Constant distance-variable time method.

4.2.1 Dispersion number , ∂

Of the two models that use dispersion number, Agunwamba model gave a higher dispersion number when compared with Leverspiel and Smith model. Hence, the higher value recorded as the dispersion coefficient using Agunwamba model. Deng *et al* model does not implore the use of dispersion number; this accounts for why it is left out of this comparison.

5 Conclusion

The calibrated dispersion coefficient values as obtained from the river gave 45 m^2/s, 42 m^2/s and 17 m$^{2/}$s for Deng *et al.*, Agunwamba and Leverspiel and smith respectively. The first two models are close indicating fitness in the observed data of the river. The disparity in the value of dispersion coefficient using Leverspiel and Smith model suggests that, the model does not fit the Mu river dispersion profile.

Finally, both Agunwamba and Deng et al models can be applied to predict the dispersion coefficient of Mu river (Natural stream).

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